Kittel TP

2.2. In the presence of a magnetic field,

$$U = -2smB \cong -\vec{M}\cdot\vec{B}.$$

Putting it into $\sigma(s) = \ln g(N,0) - 2s^2/N$, we have

$$\sigma(U) = \sigma_0 - \frac{2}{N}\left(\frac{-U}{2mB}\right)^2$$

$$= \boxed{\sigma_0 - \frac{U^2}{2Nm^2B^2}}$$

With entropy given as U, we can find τ:

$$\frac{1}{\tau} = \frac{\partial\sigma}{\partial U} = -\frac{U}{Nm^2B^2},$$

$$U = -\frac{Nm^2B^2}{\tau}$$

Note that this quantity is macroscopic, while the previous expression for $U = -2smB$ is microscopic, we can equate the two quantities by taking expectation value:

$$\langle U\rangle = -2\langle s\rangle mB = -\frac{Nm^2B^2}{\tau}.$$

$$\Rightarrow \frac{2\langle s\rangle}{N} = \frac{+\cancel{N}m^{\cancel{2}}B^{\cancel{2}}}{\tau}\frac{1}{+\cancel{mBN}} = \boxed{\frac{mB}{\tau}}$$

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